Ramsey Theory

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March 28, 2013

1. The positive integers $\{1, 2, 3, \cdots ,\}$ are coloured with k colours. Prove there exist monochromatic positive integers a, b, c such that $a + b = c$.

2. The edges of an icosahedron are coloured red and blue, such that opposite edges have the same colour. Show that there must be three edges that form a monochromatic triangle.

3. n points lie in the plane. Show that the number of unordered pairs of points X, Y such that $XY = 1$ is at most $\frac{1}{3}n^2$.

4. A planet has radius R. There are 24 GPS satellites, each of which is a distance greater than $R\sqrt{3}$ from the centre of the planet. Show that, of the $\binom{24}{2} = 276$ unordered pairs of satellites, at least 84 have a direct line of sight between the two satellites.

5. There are 10 vertices in $\mathbb{R}^3$, no four of which are coplanar. Two spiders, Archimedes and Pythagoras, each join some of the vertices with strands of web (red for Archimedes and blue for Pythagoras). Each pair of vertices can have at most one strand of web joining them, and there must not be a monochromatic triangle. What is the maximum number of edges?

6. Gottfried places 36 counters, each of which is either yellow or green, on the squares of a 6×6 chessboard. Is there necessarily a set of four counters of the same colour that form the vertices of a square?

7. (a) Does there exist a (not necessarily finite) set $S \subset \mathbb{R}^2$ of points in the plane such that $|S| \geq 2$ and for any two points $X, Y \in S$, the perpendicular bisector of XY passes through precisely two other points of S?

   $\oplus$ (b) Every point in the plane is coloured either fuchsia or lilac. Does there necessarily exist a monochromatic set S with these properties?

8. $\oplus$ Can we colour the plane with 4 colours such that there is no monochromatic continuous curve? What about with 2 colours?

* `http://cp4space.wordpress.com`